MATH 5341 - 15228 (2021 Summer)

Mathematical Modeling

Instructor: Dr. Jiwen He, jhe4@central.uh.edu.

Office Hours: By appointment.

Course Modality: Online asynchronous.

Course Platforms: MS Teams and Blackboard.

Course Technology Requirements: Computer, internet, microphone and webcam.

Textbooks: (free download)

- *Introduction to Applied Linear Algebra*, Boyd and Vandenberghe, Cambridge University Press, 2018
- *Exploring ODEs*, Trefethen, Birkisson, and Driscoll, SIAM Publishing, 2018

Prerequisites: Calculus III and Linear Algebra.

Live Online Meetings: The class will have optional live online meetings on Wednesday afternoons from 5:00 PM - 7:00 PM, starting week 2. Students are strongly encouraged to attend the live online sessions. Notes and a video of the session will be posted for asynchronous access by students.

Course Overview: The course is divided into two parts. Part I introduces vectors, matrices, and least squares methods, related topics on applied linear algebra that are behind modern data science and other applications, including document classification, prediction model from data, enhanced images, control, state estimation, and portfolio optimization. We will quickly review Part I.1 Vectors and I.2 Matrices in the first two weeks, and then focus on Part I.3 Least Squares and more advanced examples and applications in the following two and half weeks. Part II aims to use Chebfun, an open-source MATLAB package, to explore ODEs and bring new perspectives and insights on topics that are ubiquitous in advanced applications, including heat conduction, chemical reactions, chaos, population dynamics, deformations of a beam, radioactivity, bifurcation theory, stability theory, infectious diseases, nerve signals, vibrations, dynamics of networks, ballistics, planetary dynamics. Computations using Matlab will be part of regular assignments, and I will provide guidance and sample code. Students are expected to have basic familiarity with Matlab, but are not expected to know advanced Matlab programming. Students will not be tested over Matlab, but it will be necessary to use Matlab to complete many of the computations in the assignments.

Discussion Forum Activity: All students are expected to discuss the course material via the discussion forum. 10% of the final grade will be based upon discussion forum activity. Discussion forum grades will be given every 2 weeks, based upon posts from the previous 2 weeks. Questions are as important as answers, and even incorrect answers can receive points. **No Discussion Forum grades will be dropped**.

Additional Communication: Students will receive emails from the instructor a few times each week, reminding them about current material, upcoming topics, reading assignments, available help materials, and coming due dates. These emails will also be posted in a special thread within the discussion forum. Email can also be used to communicate with the instructor, although students are encouraged to use the discussion forum when the questions are not of a student specific nature.

Electronic Homework: FITB (Fill In The Blank) electronic homework will be given several times during the semester. Students can continue to submit answers after the due dates, but they will only receive 80% credit for correct answers that are late. **No FITB grades will be dropped**.

Written Assignments: Written Assignments will be given several times during the semester. Students will submit their written homework by scanning and uploading their work. Instructions will be given. **One Written Assignment will be dropped** (the one with the lowest grade).

Proctored Online Exams: A midterm and final exam will be given. At least one of these will be proctored. If an exam is not proctored, then it will be project based. Specific information will be given during the first week of class.

Grades: Discussion Forum Activity – 10%, FITBs – 15%, Written Homework – 15%, Midterm – 30%, Final Exam – 30%. Important Note: Students must score at least 50% on the final exam to pass the course (which will not be a problem for anyone keeping up and doing their own work in the course). The grading scale is standard: 94–100 (A), 90–93 (A-), 87–89 (B+), 84–86 (B), 80–83 (B-), 77–79 (C+), 74–76 (C), 70–73 (C-), 67–69 (D+), 64–66 (D), 60–63 (D-), 0–59 (F).

Whenever possible, and in accordance with 504/ADA guidelines, we will attempt to provide reasonable academic accommodations to students who request and require them.

Counseling and Psychological Services (CAPS) can help students who are having difficulties managing stress, adjusting to college, or feeling sad and hopeless. You can reach (CAPS) by calling 713-743-5454 during and after business hours for appointments or if you or someone you know is in crisis. No appointment is necessary for the "Let's Talk" program, a drop-in consultation service at convenient locations and hours around campus.

Course Topic Outline and Tentative Schedule

PART I - Exploring Linear Algebra

- Part I.1 - Vectors (1 week, June 7 - June 11)
    - Linear functions, Taylor approximation, regression model
    - Norm, distance, angle, standard deviation, and correlation
    - Clustering, the k-means algorithm, and applications
- Part I.2 - Matrices (1 week, June 14 - June 18)
    - Geometric transformations, selectors, incidence matrix, and convolution
    - Linear dynamical systems, population dynamics, epidemic dynamics, motion of a mass, supply chain dynamics
- Part I.3 - Least Squares (2.5 weeks, June 21 - July 6)
    - Least squares data fitting, validation, feature engineering
    - Least squares classification, least squares classifier, and multi-class classifiers
    - Multi-objective least squares, control, estimation and inversion, regularized data fitting
    - Constrained least squares, portfolio optimization, linear quadratic control, and linear quadratic state estimation

PART II - Exploring ODEs

- Part II.1 - Initial value problems (1 week, July 7 - July 13)
    - First-order scalar linear ODEs, smooth vs. bang-bang forcing, elimination of caffeine from the bloodstream
    - First-order scalar nonlinear ODEs, scalarization by complex arithmetic, classic pursuit problems
    - Second-order ODEs and damping, elliptical/non-elliptical orbits, skydiver
- Part II.2 - Boundary value problems (1 week, July 14 - July 20)
    - Boundary-value problems, side conditions, beam theory and the strength of spaghetti
    - Eigenvalues of linear BVPs, eigenstates of the Schrodinger equation
    - Variable coefficients and adjoints, automated computation of adjoints, adjoints and optimization
- Part II.3 - Exploring ODEs (3 weeks, July 21 - August 10)
    - Resonance, periodic forcing gives periodic solutions, moon, sun, and tides
    - Second-order equations in the phase plane, BVPs as well as IVPs, nonlinear pendulum
    - Systems of equations, SIR epidemiology models

- Random functions and random ODEs, stochastics via smooth functions, metastability, radioactivity, and tunneling
- Chaos, transient chaos in the 3-body problem, chaos in a food web
- Linearization, linearized Lorenz trajectories
- Stable and unstable fixed points, application to transition to turbulence, transition to turbulence in a pipe
- Multiple solutions of nonlinear BVPs, shooting to find multiple solutions, sending a spacecraft to a destination
- Bifurcation, tracking hysteresis as a parameter varies, FitzHugh–Nagumo equations of neural signals
- Continuation and path-following, Arrhenius chemical reaction