Partial Differential Equations

Math 6326, Fall 2017.

G. Auchmuty auchmuty@uh.edu

This course will provide an introduction to the modern theory of elliptic partial differential equations using Sobolev space methods. The prerequisite is competence in multivariable calculus and real analysis. Ideally a student should have done well in M6320-21 and having a working knowledge of Lebesgue integration and some Fourier analysis. The basic constructions of linear analysis in Hilbert and Banach spaces will be assumed known.

The first lectures will provide an introduction to boundary value problems for elliptic PDEs and describe the types of results that will be obtained. In particular weak formulations of various boundary problems and the background to finite element modeling of the problems will be described. Some results about the formulations typically used for numerical simulations will be treated in the course.

To do this the calculus of weak derivatives and the associated Sobolev function spaces needed for weak formulations of boundary value problems will be studied. Various results needed for the existence and properties of solutions of partial differential equations will be proved. There will be an emphasis on important examples from applications involving equations posed on bounded subsets of $\mathbb{R}^N$ with $1 \leq N \leq 3$.

The instructor will provide notes on much of the material for the course and there is no prescribed text. The course will cover the material in the first three chapters of "Hilbert Space Methods in Partial Differential Equations" by Ralph E. Showalter (Dover or free online) and some of the text "Elliptic Equations: An Introductory Course",by Michel Chipot, Birkhauser, 2009. The Universitext "Functional Analysis, Sobolev Spaces and Partial Differential Equations" by Haim Brezis, Springer 2011 provides a thorough treatment of the functional analysis used. These three texts may be good reference texts for the material treated in the course.