Enclosing Hybrid Behavior

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The Effective Modeling Group (EMG):

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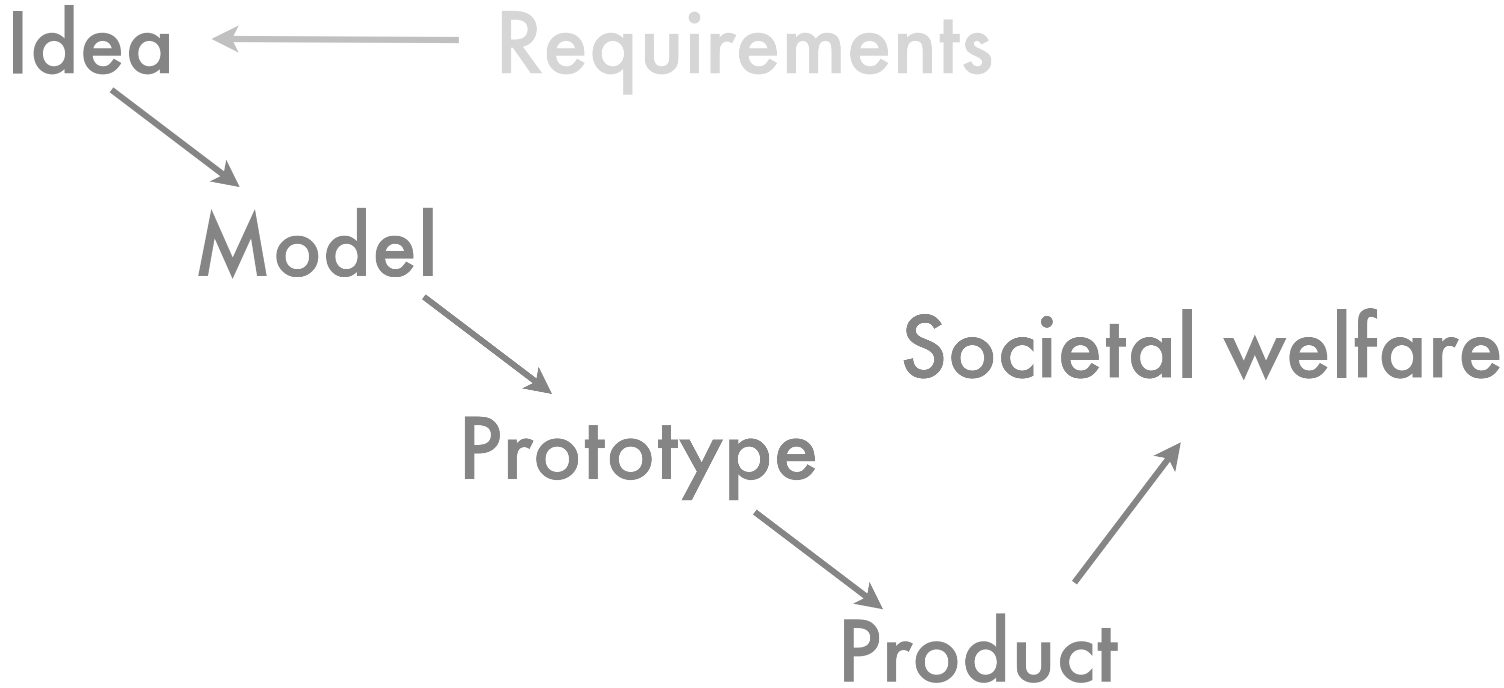
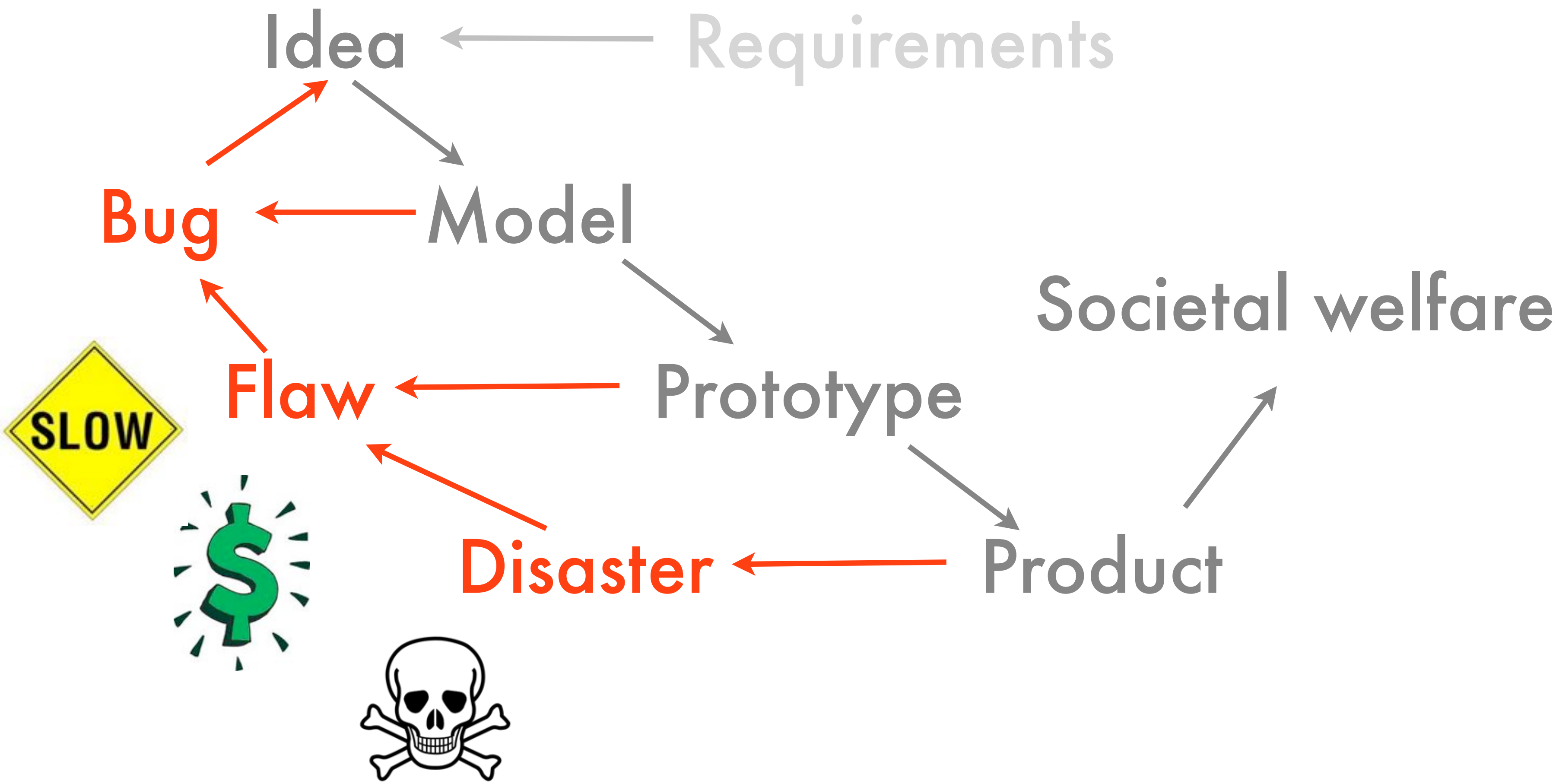
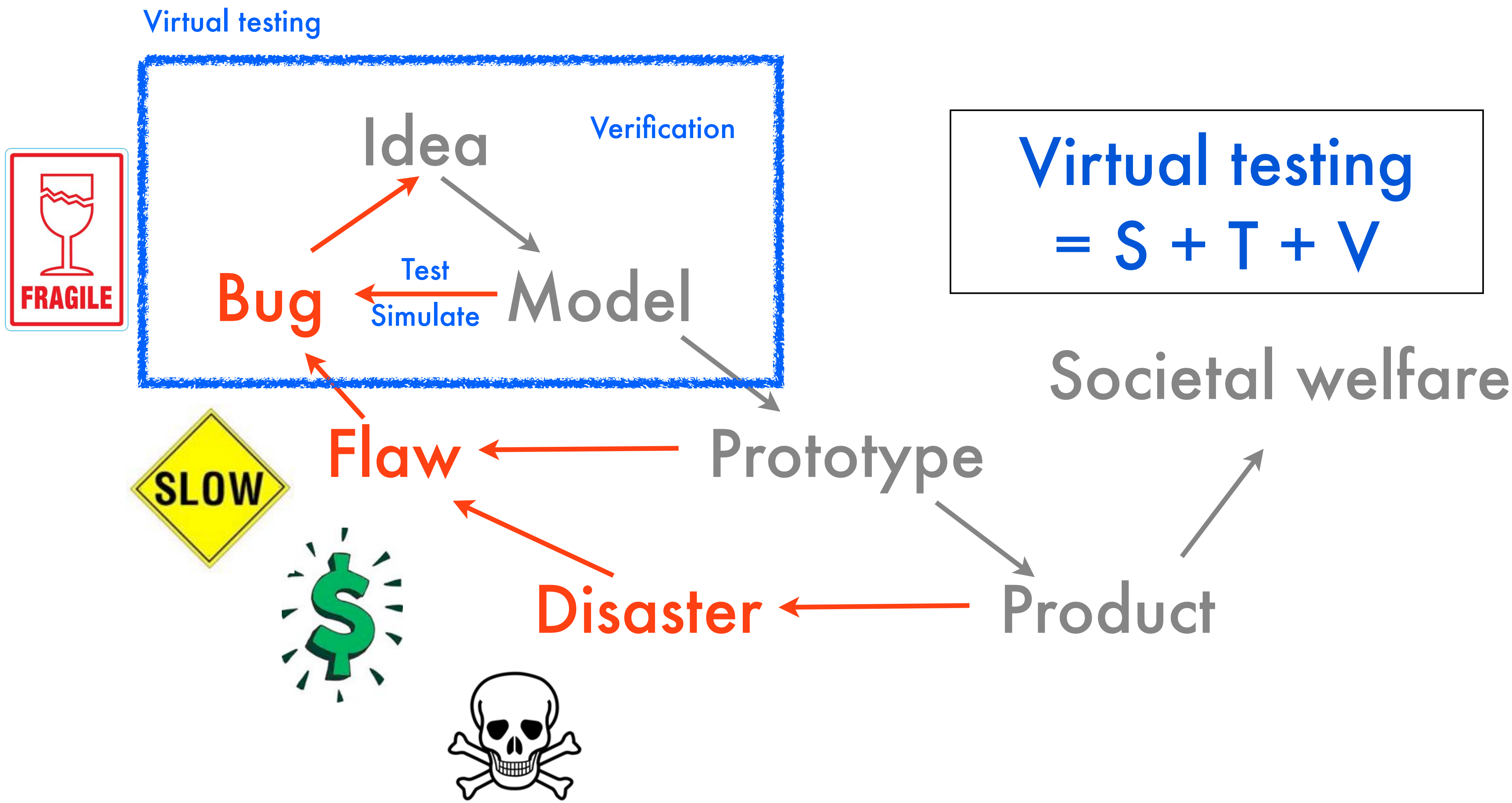
What is innovation?



Innovation theory

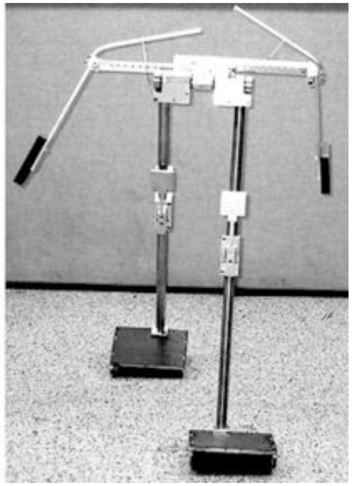
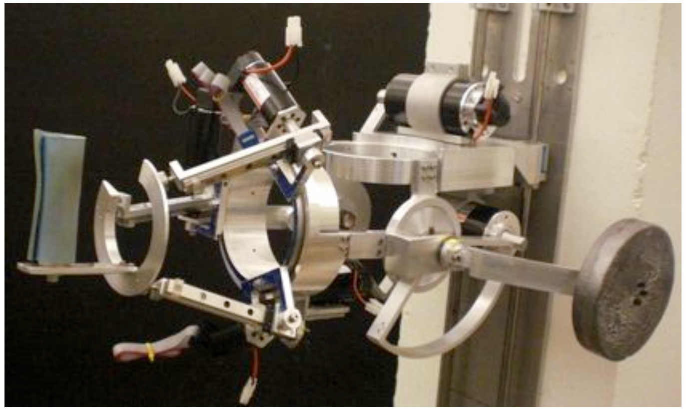


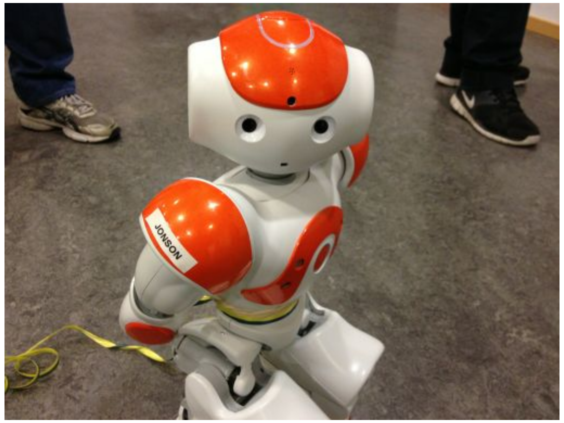
Virtual testing



Virtual testing
= S + T + V

Robot design



Simulation tools today

- No guarantee that behavior computed is consistent with model used.
  - Numerical artifacts
  - Integration drift
  - Singularities often ignored
  - Zeno behavior

Rest of this talk

- Enclosure methods
- Enclosing continuous behaviors
- Enclosing hybrid systems
  - Event detection and reset maps
  - Zeno behavior
- Conclusions

Idea: Enclosure methods

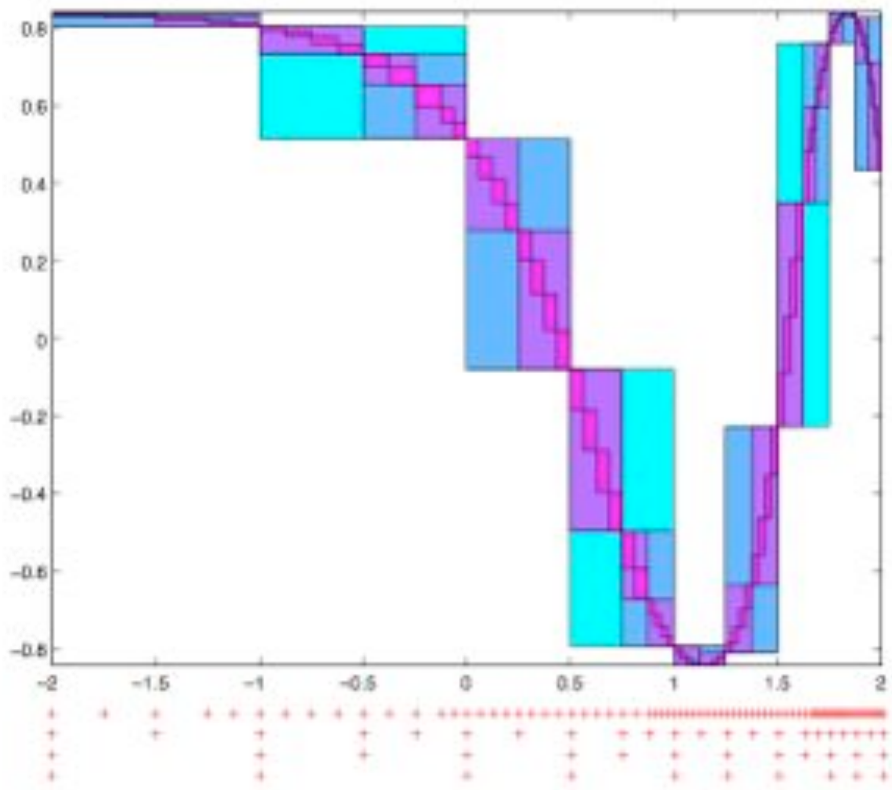


Figure 5.14: Refined enclosures of the integrand $f(x) = \sin(\cos(e^x))$.

- Always guarantee that solution is enclosed
- Can compute more precise answers as needed
- But can they be mechanized?

Continuous behaviors

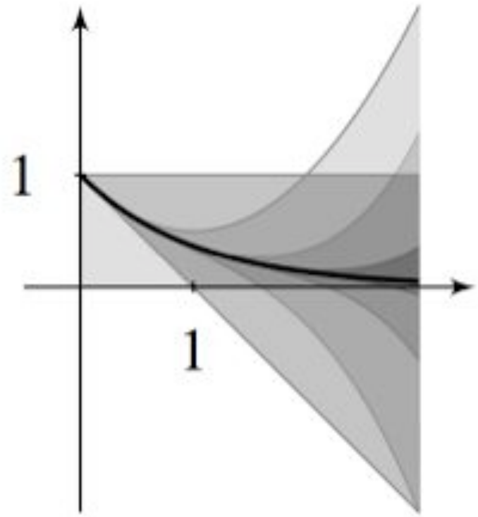
- An elegant, very general method exists:
  - Picard iteration
- Key challenge: Extending to proper enclosures

Set

$$\varphi_0(t) = y_0$$

and

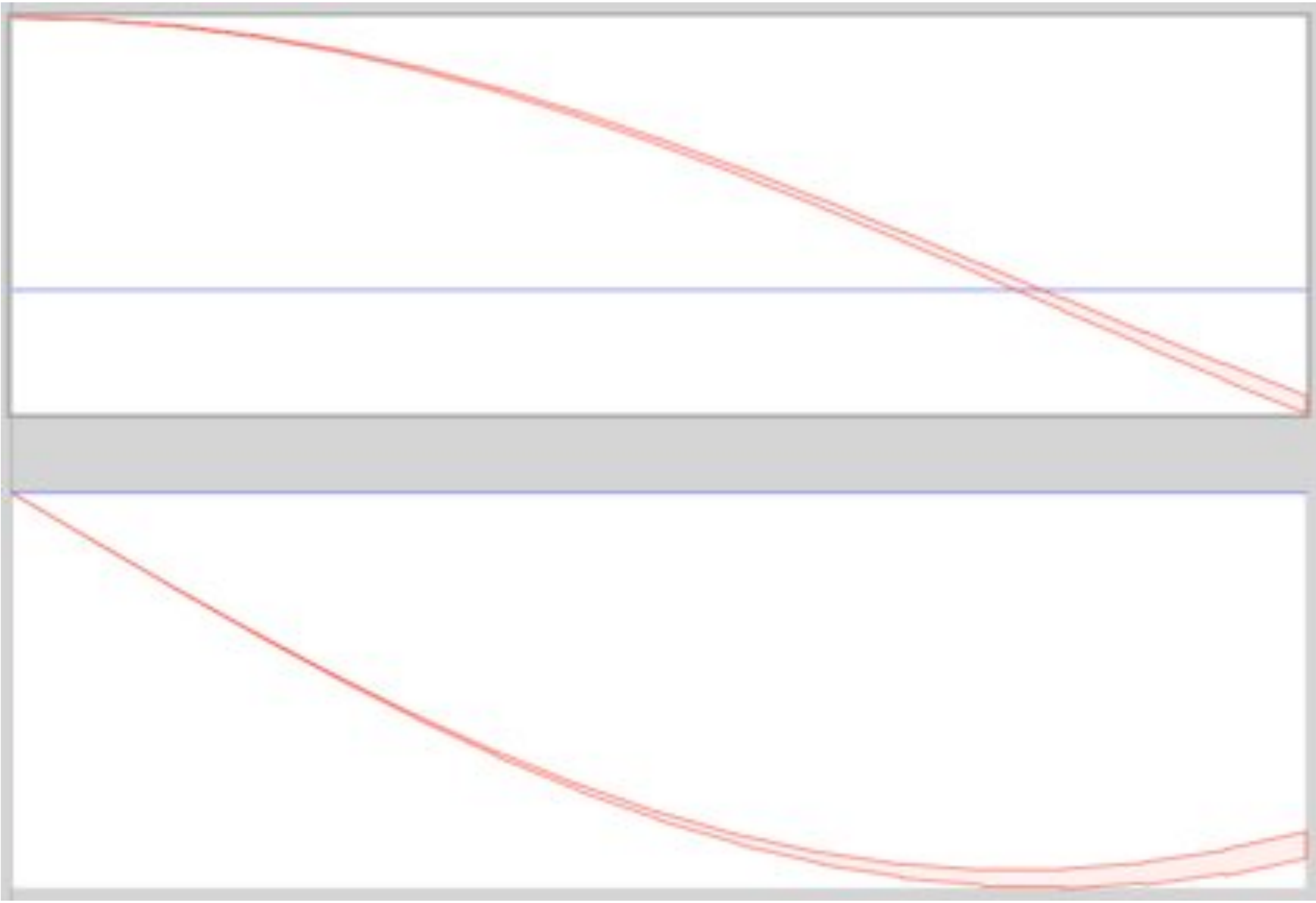
$$\varphi_{k+1}(t) = y_0 + \int_{t_0}^{t} f(s, \varphi_k(s))\, ds.$$



Example

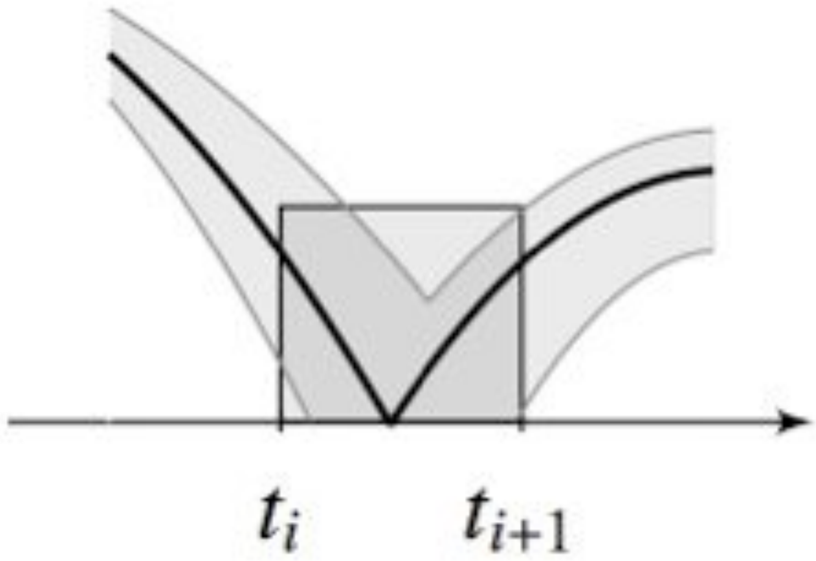
```
class Main(simulator)
private x:=1; x':=0; x'':=0; mode := ""; end
  switch mode
    case ""
      x'' = -x
  end;
  simulator.endTime := 2.0;
  simulator.minSolverStep := 0.1;
end
```

Example



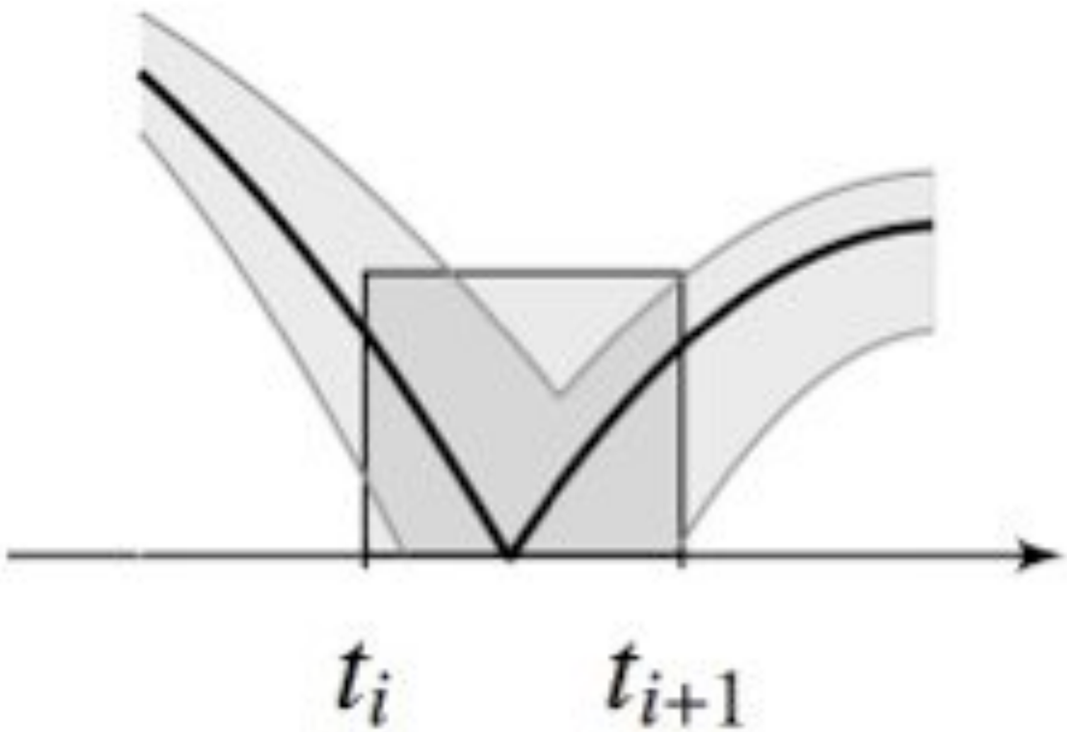
Event detection

- Enclosures provide a natural method for event detection (root find)
- Basic idea:
  - Mean value theorem
  - It's OK to say "I don't know"



Reset maps

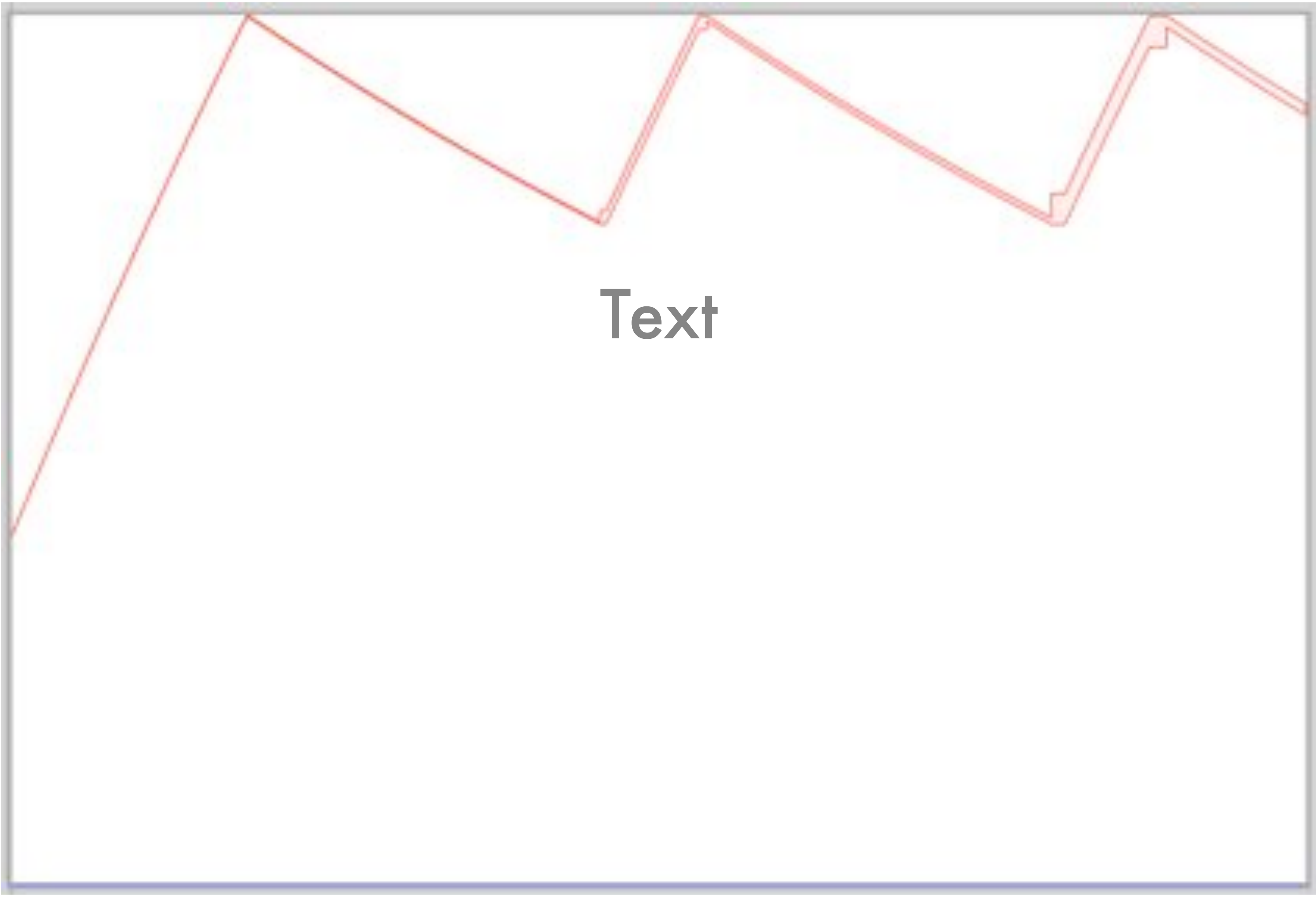
- Assume worst case behavior
- Note: Still need to know it was only *one* event that occurred in that interval



Example

```
class Main (simulator)
  private
    mode := "on"; x := 10; x' := 0;
  end
  switch mode
    case "on" require x <= 25
      if x == 25
        mode := "off"
      end;
      x' = 100 - x;
    case "off" require x >= 19
      if x == 19
        mode := "on"
      end;
      x' = - x;
  end;
  simulator.endTime := 1;
  simulator.minSolverStep := 0.01;
  simulator.minLocalizationStep := 0.001;
  simulator.minComputationImprovement := 0;
end
```

Example

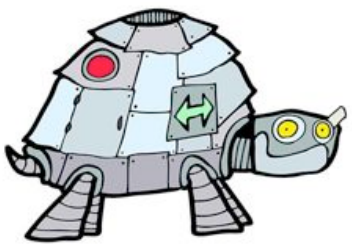
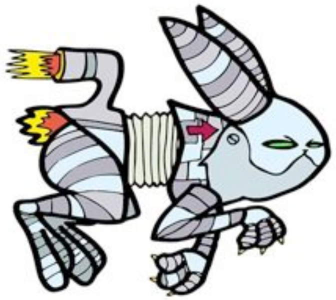
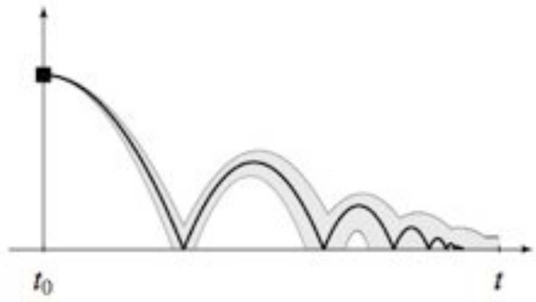


A bouncing ball

```
class Main(simulator)
  private
    mode := "Fly";
    x    := 5;
    x'   := 0;
    x''  := 0;
  end
  switch mode
    case "Fly"
      if x == 0 && x' <= 0
        x' := -0.5*x';
        mode := "Fly";
      end;
      x'' = -10;
  end;
  simulator.endTime := 4.5;
  simulator.minSolverStep := 0.01;
  simulator.minLocalizationStep := 0.01;
  simulator.minComputationImprovement := 0.001;
end
```

Zeno Behavior

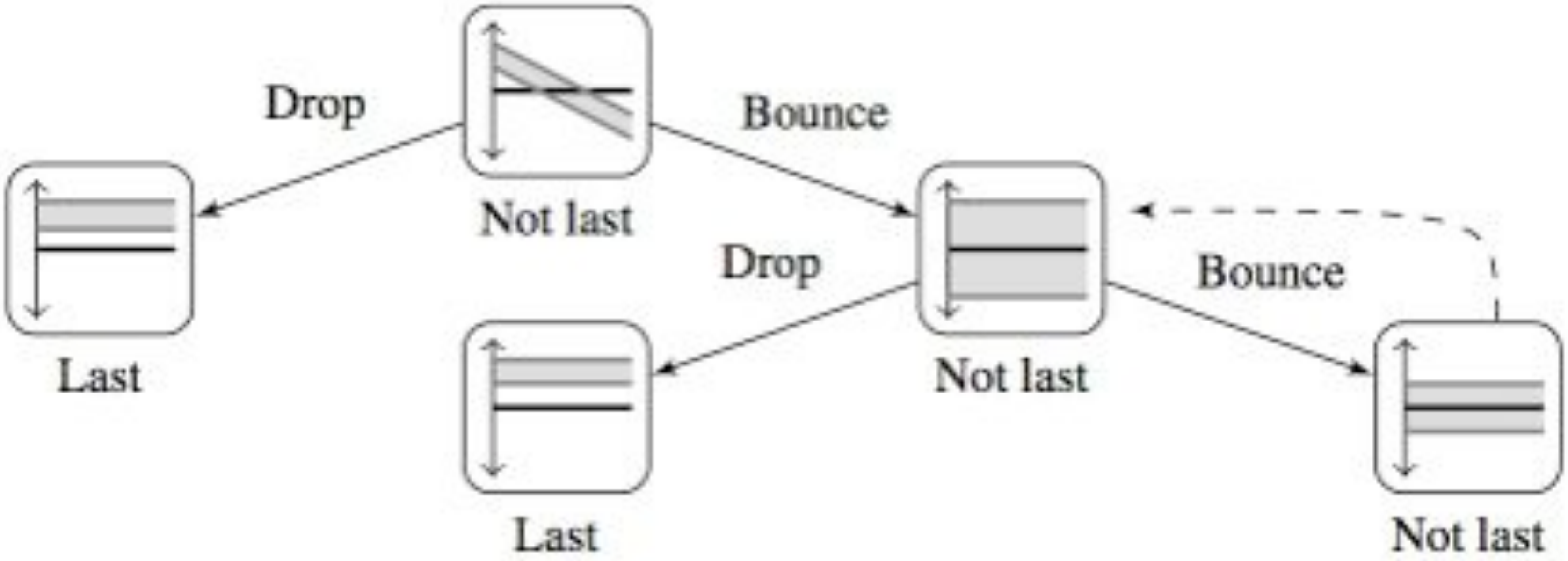
- A real problem for rigid body dynamics *with impacts*

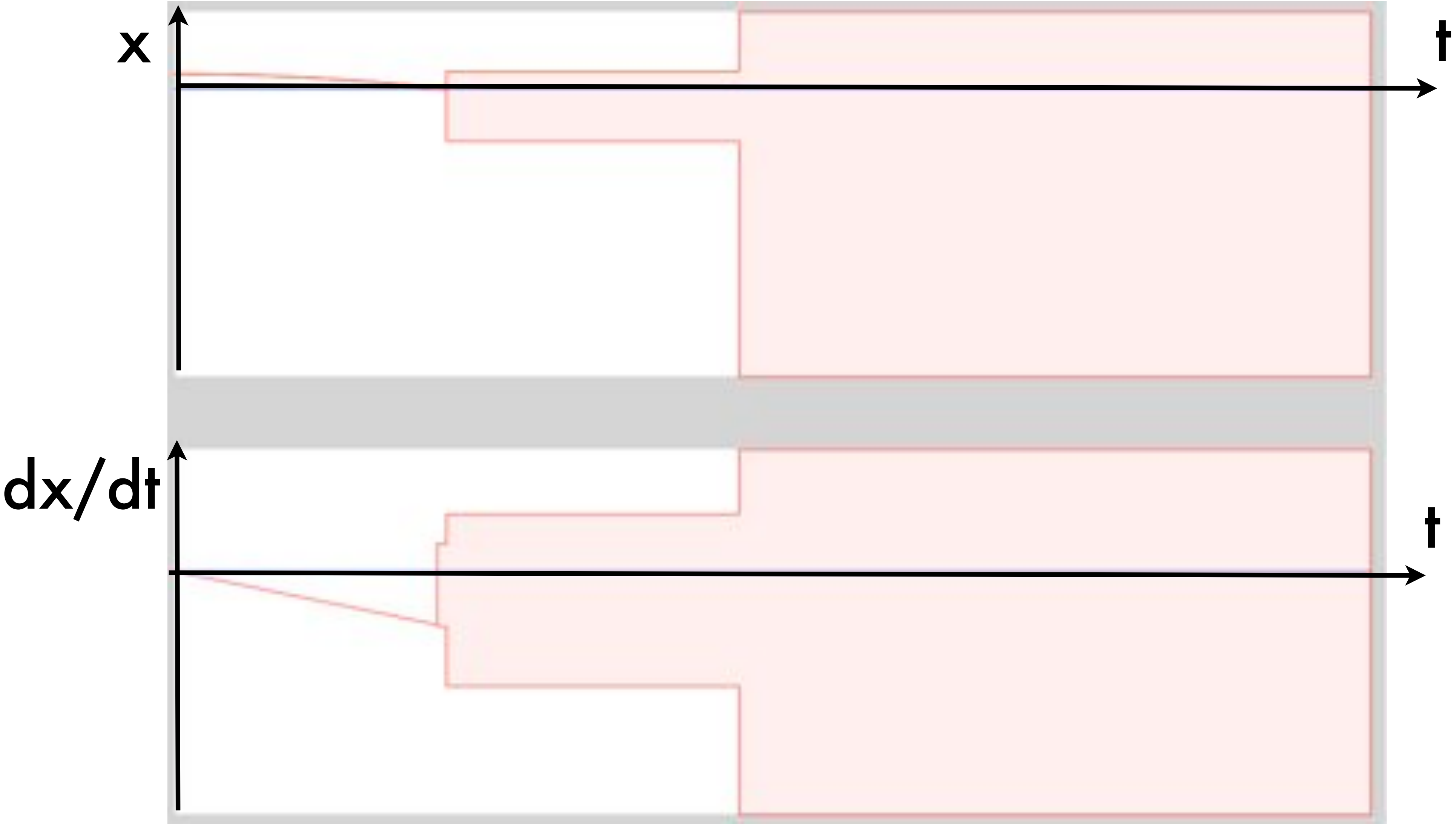
- A bouncing ball comes to rest in finite time, but it does so with an *infinite* number of bounce events!

Enclosing Zeno

- Idea: We can actually relax that requirement if we know that a repeat event does NOT enlarge the enclosure we start with



Enclosing Zeno, Take I

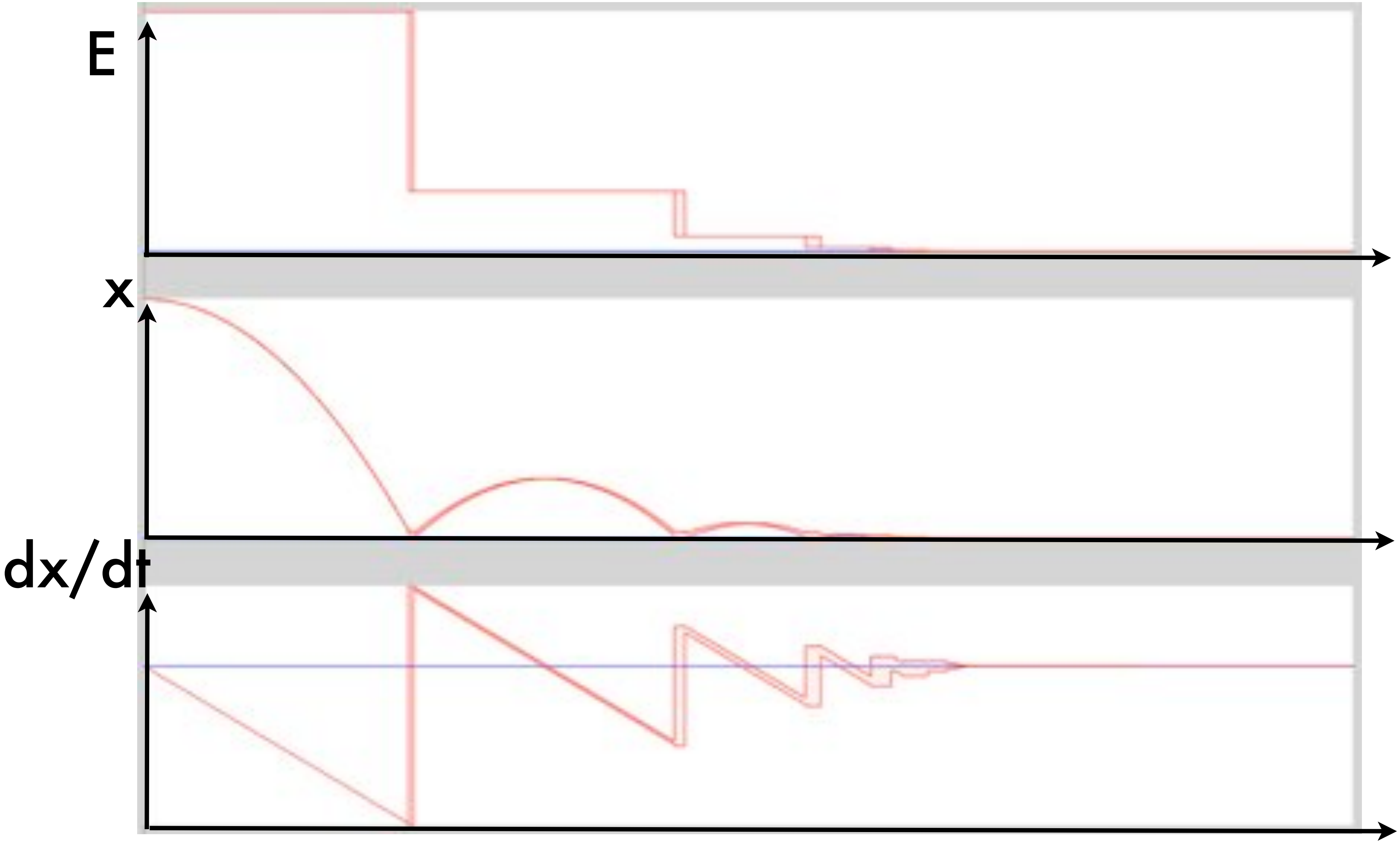


Fix: Over-constraining

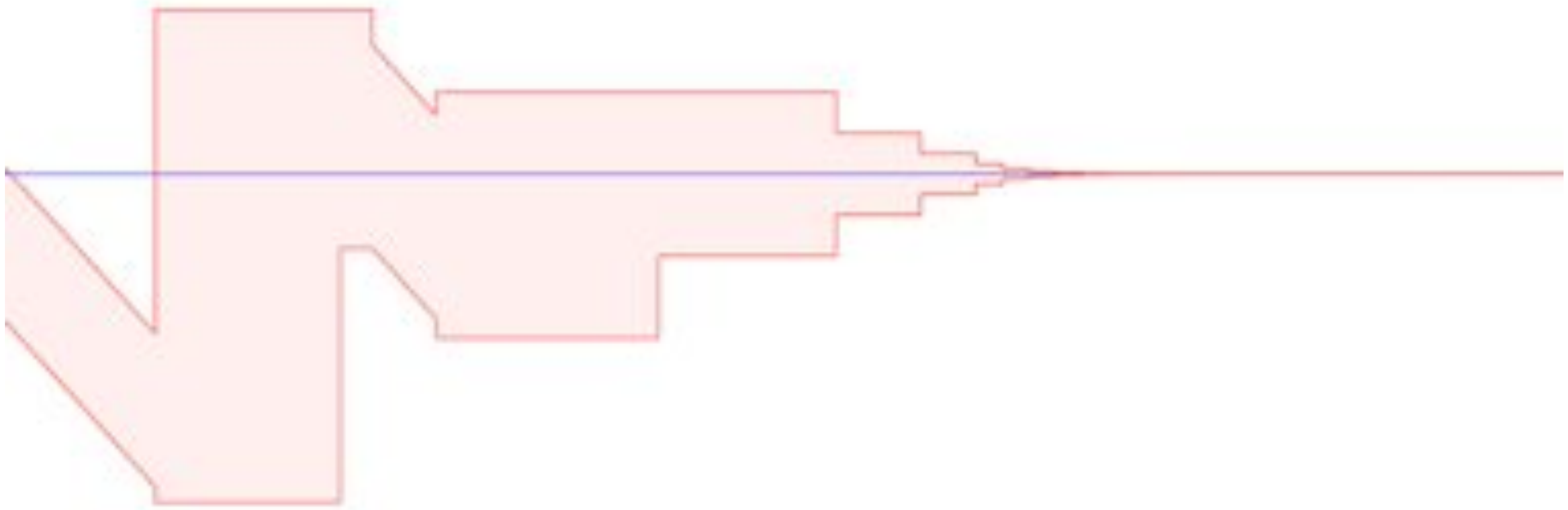
- Enforce domain constraints (intersect)
    - Example: $x >= 0$
- Constraining speed based on explicit energy
    - Example: A notion of energy

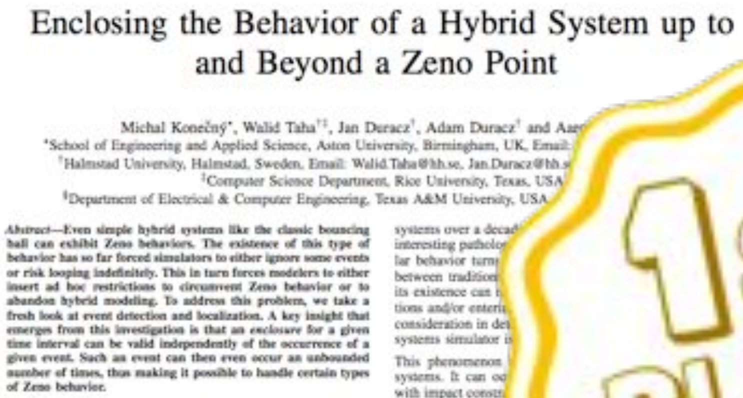
Enclosing Zeno, Take II

Enclosing Zeno, Take III



Empire State Building





Enclosing the Behavior of a Hybrid System up to and Beyond a Zeno Point

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Abstract—Even simple hybrid systems like the classic bouncing ball can exhibit Zeno behaviors. The existence of this type of behavior has so far forced simulators to either ignore some events or risk looping indefinitely. This in turn forces modelers to either insert ad hoc restrictions to circumvent Zeno behavior or to abandon hybrid modeling. To address this problem, we take a fresh look at event detection and localization. A key insight that emerges from this investigation is that an *enclosure* for a given time interval can be valid independently of the occurrence of a given event. Such an event can then even occur an unbounded number of times, thus making it possible to handle certain types of Zeno behavior.

systems over a
interesting
lar behavior
between
its existence can
tions and/or
consideration in
systems simulator

This phenomenon
systems. It can
with impact

Technical results

- Proper interval Picard converges
- Event detection is sound
- Zeno method is sound

Next Generation Testing

Uncertainty-aware design

Activity in NG-Test

- Analysis of ISO 26262-3
- Defining high-level models of test scenarios
  - Vehicles, controls, sensors
- Using enclosures to establish bounds on severity of collisions
- Gradual model refinement is key

Conclusions

- Using enclosures
    - ensures that any answer produced is correct
    - simplifies correct event detection
    - admits an elegant way of handling certain classes of Zeno behavior
    - benefits from over-constraining

Future work

- Understanding algorithmic complexity
- Understanding performance on larger models (mainly drawn from the robotics domain)
- Identifying heuristics to limit loss of precision during continuous segments

Thank you!

- Checkout acumen-language.org